Lorentz Violation, Electrodynamics, and the Cosmic Microwave Background*

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Vacuum birefringence is a signature of Lorentz-symmetry violation. Here we report on a recent search for birefringence in the cosmic microwave background. Polarization data is used to place constraints on certain forms of Lorentz violation.

I. INTRODUCTION

The properties of light have proved to be a valuable testing ground for special relativity for more than a century. Contemporary experiments are motivated in part by a possible breakdown of special relativity with origins in Planck-scale physics [1, 2, 3]. These experiments include modern versions of the classic Michelson-Morley and Kennedy-Thorndike experiments that use highly stable resonant cavities to search for violations of rotation and boost symmetries [4]. However, the highest sensitivities to relativity violations in electrodynamics are found in searches for vacuum birefringence in light from very distant sources [5, 6, 7, 8]. Birefringence studies take advantage of the extremely long baselines that allow the miniscule effects of a Lorentz violation to accumulate to (potentially) detectable levels over the billions of years it takes for the light to reach Earth. The cosmic microwave background (CMB) is the oldest light available to observation and therefore provides an excellent source for birefringence searches. Here we summarize a recent search for signals of Lorentz violation using CMB polarimetry [8].

General Lorentz violation is described by a framework known as the Standard Model Extension (SME) [3]. The SME provides the theoretical backbone for studies in a number of areas [2], including photons [4, 6, 7, 8, 9]. Most tests of Lorentz violation focus on the minimal SME, which assumes usual gauge invariance and energy-momentum conservation and restricts attention to superficially renormalizable operators. Operators of dimension $d \leq 4$ are of renormalizable dimension and are included the minimal SME. Two types of Lorentz-

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violating operators appear in the minimal SME, CPT-odd operators with coefficients $(k_{AF})_\kappa$ and CPT-even operators with coefficients $(k_F)^{\kappa\lambda\mu\nu}$.

In this work, we also consider non-minimal higher-dimensional operators in the photon sector with $d > 4$. In general there are an infinite number of possible operators that emerge when we relax the renormalizable condition. These operators are phenomenologically and theoretically relevant in that they help provide a connection to the underlying Planck-scale physics. They also add a number of new and interesting signals for Lorentz violation that may be tested experimentally.

II. THEORY

General Lorentz-violating electrodynamics is given by a lagrangian that takes the same basic form as the minimal-SME photon sector [8]:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\epsilon^{\kappa\lambda\mu\nu}A_\lambda(\hat{k}_{AF})_\kappa F^{\mu\nu} - \frac{1}{4}F_{\kappa\lambda}(\hat{k}_F)^{\kappa\lambda\mu\nu}F_{\mu\nu} \ . \tag{1}$$

We assume a linear theory and impose the usual U(1) gauge invariance. The key difference between this theory and the minimal-SME photon sector is that here the $\hat{k}_{AF}$ and $\hat{k}_F$ coefficients are differential operators. The effects of these operators mimic the effects of a permeable medium whose activity depends on the photon energy and momentum. This introduces a plethora of new effects not found in either the conventional Lorentz-conserving case or the minimal SME. These include drastically different frequency dependences and direction-dependent propagation of light.

Expanding the $\hat{k}_{AF}$ and $\hat{k}_F$ operators in the 4-momentum $p_\mu = i\partial_\mu$ leads to the expressions

$$(\hat{k}_{AF})_\kappa = \sum (k_{AF}^{(d)})_\kappa{}^{\alpha_1\ldots\alpha_{(d-3)}}\partial_{\alpha_1}\ldots\partial_{\alpha_{(d-3)}} \ , \tag{2}$$

$$(\hat{k}_F)^{\kappa\lambda\mu\nu} = \sum (k_F^{(d)})^{\kappa\lambda\mu\nu\alpha_1\ldots\alpha_{(d-4)}}\partial_{\alpha_1}\ldots\partial_{\alpha_{(d-4)}} \ . \tag{3}$$

The coefficients for Lorentz violation associated with the dimension d operators are now given by $(k_{AF}^{(d)})_\kappa{}^{\alpha_1\ldots\alpha_{(d-3)}}$ and $(k_F^{(d)})^{\kappa\lambda\mu\nu\alpha_1\ldots\alpha_{(d-4)}}$. The $\hat{k}_{AF}$ expression contains all CPT-breaking effects, and the sum is restricted to odd-dimensional operators, $d =$ odd. The $\hat{k}_F$ coefficients control all CPT-even violations and have $d =$ even. Imposing gauge invariance places various constraints on these coefficients. For $k_{AF}^{(d)}$ coefficients, any trace of the that involves the first index vanishes identically. For $k_F^{(d)}$, the antisymmetrization on any three indices

vanishes. Standard group theory techniques allow a counting of the independent coefficients for Lorentz violations [10]. For a given dimension d, we find $\frac{1}{2}(d+1)(d-1)(d-2)$ independent $k_{AF}^{(d)}$ coefficients in the CPT-odd case and $(d+1)d(d-3)$ independent $k_F^{(d)}$ coefficients in the CPT-even case.

For studies of Lorentz-violation induced birefringence, certain linear combinations of these general coefficients are important. They result from a spherical-harmonic expansion of plane-waves propagating in the vacuum. This plane-wave expansion is best characterized using the language of Stokes parameters. We begin by defining a Stokes vector $\boldsymbol{s} = (s^1, s^2, s^3)^T$. The direction in which this vector points in the abstract 3-dimensional Stokes space uniquely characterizes the polarization of the radiation. Stokes vectors lying in the s^1-s^2 plane correspond to all possible linear polarizations, while Stokes vectors parallel and antiparallel to the s^3 axis give the two circular polarizations. General right-handed elliptical polarizations point in the upper-half Stokes space, $s^3 > 0$, while left-handed are given by the lower half, $s^3 < 0$.

This formalism provides an intuitive picture of birefringence. It can be shown that birefringence causes a rotation of the Stokes vector $\boldsymbol{s}$ about some axis $\boldsymbol{\varsigma}$. This occurs whenever the usual degeneracy between the various polarizations in broken. Formally, we solve the modified equations of motion. We find that some types of violations lead to two propagating eigenmodes that have slightly different velocities. They also differ in polarization, and light of an arbitrary polarization is a superposition of the two eigenmodes. This superposition is altered as the eigenmodes propagate at different velocities, causing an oscillatory effect that reveals itself as a rotation of the Stokes vector. The rotation takes the form

$$d\boldsymbol{s}/dt = 2\omega\boldsymbol{\varsigma} \times \boldsymbol{s} \ , \tag{4}$$

where ω is the wave frequency, and the rotation axis $\boldsymbol{\varsigma}$ corresponds to the Stokes vector of the faster eigenmode. In general, $\boldsymbol{\varsigma}$ may depend on both the direction of propagation and the frequency.

The basic idea of a birefringence test is to examine light from a distant polarized source for the above rotation. To do this we need to express the rotation axis $\boldsymbol{\varsigma}$ in terms of the coefficients for Lorentz violation. The general result is rather complicated, but can be written in a relatively simple form in terms of a set of "vacuum" coefficients, which are linear combinations of the general coefficients. The calculation involves decomposing $\boldsymbol{\varsigma}$ into

spin-weighted spherical harmonics. The result takes the form

$$\varsigma^1 \mp i\varsigma^2 = \sum_{dlm} \omega^{d-4}(k^{(d)}_{(E)lm} \pm ik^{(d)}_{(B)lm})\ {}_{\pm2}Y_{lm}(\hat{\boldsymbol{n}}) \ , \tag{5}$$

$$\varsigma^3 = \sum_{dlm} \omega^{d-4} k^{(d)}_{(V)lm}\ {}_0Y_{lm}(\hat{\boldsymbol{n}}) \ , \tag{6}$$

where ${}_sY_{lm}$ is a spin-weighted spherical harmonic with spin-weight s, and $\hat{\boldsymbol{n}}$ is the radial unit vector pointing toward the source on the sky. The vacuum coefficients $k^{(d)}_{(V)lm}$, $k^{(d)}_{(E)lm}$, and $k^{(d)}_{(B)lm}$ represent the minimal combinations of coefficients for Lorentz violation that cause birefringence and affect polarization. The designations E and B refer to the parity of the coefficient and is borrowed from radiation theory. In the next sections, we describe a search for these effects in existing CMB polarization data.

III. CMB

The CMB is conventionally parameterized by a spin-weighted spherical-harmonic expansion similar to the expansion of $\boldsymbol{\varsigma}$ given above [11, 12]. The complete characterization of radiation from a given point on the sky includes the temperature T, the linear polarization, given by Stokes parameters s^1 and s^2, and the circular polarization, given by s^3. The global description is given by the expansion

$$T = \sum a_{(T)lm}\ {}_0Y_{lm}(\hat{\boldsymbol{n}}) \ , \qquad s^3 = \sum a_{(V)lm}\ {}_0Y_{lm}(\hat{\boldsymbol{n}}) \ ,$$
$$s^1 \mp is^2 = \sum (a_{(E)lm} \pm ia_{(B)lm})\ {}_{\pm2}Y_{lm}(\hat{\boldsymbol{n}}) \ . \tag{7}$$

One then constructs various power spectra,

$$C_l^{X_1X_2} = \tfrac{1}{2l+1} \sum_m \langle a^*_{(X_1)lm} a_{(X_2)lm} \rangle \ , \tag{8}$$

where $X_1, X_2 = T, E, B, V$. These spectra quantify the angular size variations in each mode and any correlation between different modes. Smaller l correlates to larger angular size on the sky.

Within conventional physics, we expect a nearly isotropic ($l = 0$) temperature distribution. However, tiny fluctuations in temperature during recombination not only introduce higher-order multipole moments ($l > 0$) but also provide the necessary anisotropies to produce a net polarization. Only linear polarizations are expected since no circular polarization

is produced in Thomson scattering. Furthermore, E-type polarization is expected to dominate and be correlated with the temperature. No correlation is expected between the much smaller B polarization and temperature. This general picture agrees with observation to the extent to which CMB radiation has been measured [13, 14].

A breakdown of Lorentz symmetry may alter these basic features. Some of the new effects can be readily understood as consequences of the Stokes rotations. For example, the CPT-odd coefficients $k^{(d)}_{(V)lm}$ lead to a Stokes rotation axis that points along the s^3 direction. The resulting local rotations in polarization leave the circularly polarized component unchanged. However, it does lead to a rotation in the linear components, causing a simple change in the polarization angle at each point on the sky. Globally this causes a mixing between the E and B polarization. This could introduce an unusually large B component, which gives a potential signal of CPT and Lorentz violation. Similar mixing can arise from the $k^{(d)}_{(E)lm}$ and $k^{(d)}_{(B)lm}$ coefficients. However, since these give a rotation axis that lies in the s^1-s^2 plane, the rotations in this case also introduce circular polarization. So a large circularly polarized component in the CMB might indicate a CPT-even violation of Lorentz invariance.

All except the $d = 3$ coefficients result in frequency-dependent rotations. Also, only the $l = 0$ coefficients cause isotropic rotations that are uniform across the sky. As a result, the coefficient $k^{(3)}_{(V)00}$ provides a simple isotropic frequency-independent special case. A calculation shows that this case causes a straightforward rotation between C_l^{EE}, C_l^{BB}, and C_l^{EB}, as well as between C_l^{TE} and C_l^{TB} [15]. In contrast, more general anisotropic and frequency-dependent cases cause very complicated mixing between the various $C_l^{X_1X_2}$ and require numerical integration of the rotation (4) over the sky and frequency range.

IV. RESULTS

To illustrate the kinds of sensitivities that are possible in CMB searches for birefringence, we next examine the results of the BOOMERANG experiment [14]. This balloon-based experiment made polarization measurements in a narrow band of frequencies at approximately 145 GHz. This relatively high frequency implies that BOOMERANG is well suited to birefringence tests since for all violations, except those with $d = 3$, higher photon energy implies a larger rotation in polarization. The small frequency range is also helpful since we can approximate all frequencies as ~ 145 GHz.

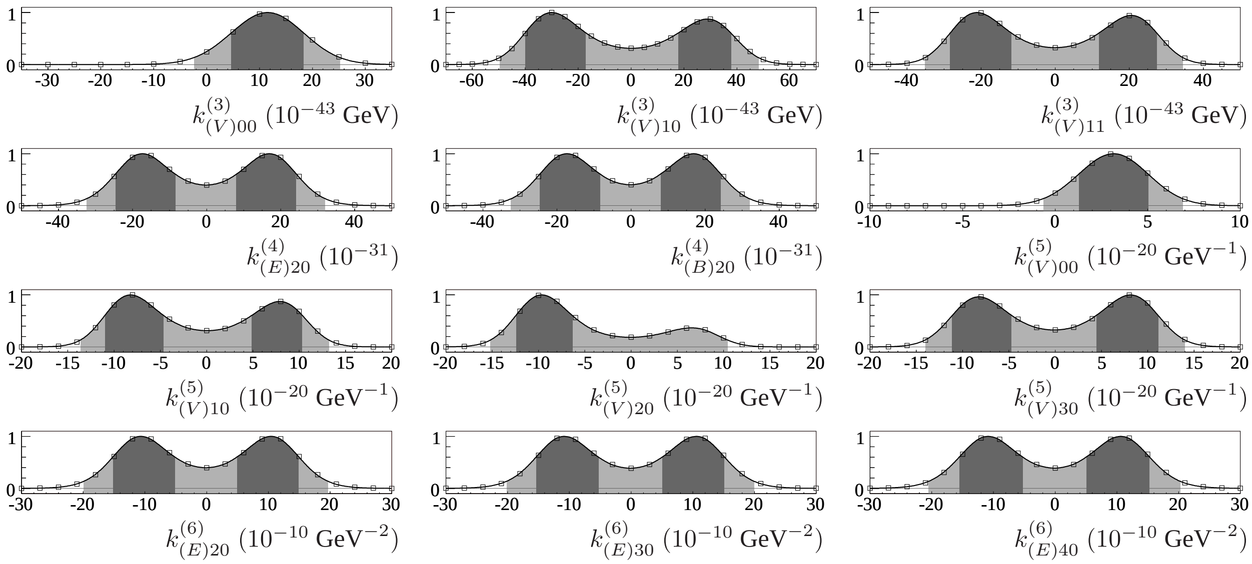

FIG. 1: Relative likelihood versus coefficients for Lorentz violation. Boxes indicate numerically calculated values, and the curve is the smooth extrapolation of these points. The dark-gray shaded region indicates the 68% confidence level, and the light-gray shows the 95% level.

In our calculation, we assume conventional polarization is produced during recombination and numerically determine the rotated polarization for points across the sky. The resulting $C_l^{X_1X_2}$ for various values of coefficients for Lorentz violation are determined and compared to published BOOMERANG results. Figure 1 shows the calculated relative likelihood for a sample of 12 coefficients for Lorentz violation. In each case, we vary the value of one coefficient, setting all other coefficients to zero. The 1σ and 2σ regions are shown.

Some generic features are seen in our survey. In each case, the coefficient is nonzero at the 1σ level, hinting at possible Lorentz violation. However, since this occurs in every case, it is likely that this indicates some systemic feature of the BOOMERANG data or our analysis. We also see that each case is consistent with no Lorentz violation at the 2σ level, giving conservative upper bounds on the 12 coefficients in Figure 1.

These results demonstrate the potential of the CMB for testing Lorentz invariance. Due to the long propagation times, the sensitivities to $d = 3$ coefficients afforded by the CMB are near the limit of what can be expected in birefringence tests. However, for $d \geq 4$, better sensitivities might be obtained using high-frequency sources like gamma-ray bursts [7]. Regardless, because of its all-sky nature, the CMB provides a useful probe that can simultaneously probe large portions of coefficient space, which is difficult in searches involving a

handful of point sources that access a limited number of propagation directions.

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